The design of controllers that operate in dynamical systems to perform specified tasks has traditionally been manual. Machine learning algorithms enable data-driven generation of controllers, also called policies or programs, and differ in how a user may convey what task the controller should perform. In Imitation Learning (IL), the user demonstrates a supervisor control signal in a set of execution traces, and the objective is to train from this data a controller that performs the computation correctly on unseen inputs.

This paper takes a hierarchical imitation learning approach to program synthesis. We model the controller as a set of Parametrized Hierarchical Procedures (PHPs) (Fox et al., 2018), each of which can invoke a sub-procedure, take a control action, or terminate and return to its caller. The PHP model maintains a similar call-stack to that of Neural Programmers–Interpreters (NPI) (Reed & De Freitas, 2015; Fox et al., 2018), each of which can invoke a sub-procedure, take a control action, or terminate and return to its caller. The PHP model with procedures that take arguments; (2) a hierarchical variational inference method for training PHPs from weak supervision. Our method generalizes Stochastic Recurrent Neural Networks (SRNNs) (Fraccaro et al., 2016) to hierarchical controllers. Compared to level-wise hierarchical training via the Expectation–Gradient (EG) method (Fox et al., 2018), our SVI approach applies to deeper hierarchies and to procedures that take arguments.

Hierarchical Variational Inference. A Partially Observable Markov Decision Process (POMDP) with states \(s_t \in \mathcal{S}\), observations \(o_t \in \mathcal{O}\), and actions \(a_t \in \mathcal{A}\) has dynamics \(p(s_{t+1}, o_{t+1}|s_t, a_t)\). A controller with memory state \(m_t \in \mathcal{M}\) has policy \(p_{\theta}(m_{t+1}, a_t|m_t, o_t)\).

We consider weakly supervised demonstrations, which are observation–action traces \(\xi = (o_0, a_0, \ldots, o_{T-1}, a_{T-1})\), and strong supervision further augmented by the trajectory of internal agent states \(\zeta = (m_0, \ldots, m_{T-1})\). In our extension of the Parametrized Hierarchical Procedures (PHP) model (Fox et al., 2018), the memory state is a call-stack \(m = [(h_0, u_0, \tau_0), \ldots, (h_d, u_d, \tau_d)]\), each frame in the stack consisting of the identifier \(h \in \mathcal{H}\) of a PHP, its argument \(u \in \mathcal{U}\), and its program counter \(\tau \in \mathbb{Z}^+\). A PHP is a function \(\pi^h : (u, \tau, o) \mapsto \text{operation}\), represented by a neural network. We repeatedly take a step of the top PHP \(h_d\) with inputs \((u_d, \tau_d, o_t)\), deciding to either (1) terminate and be popped from the stack; (2) call a sub-procedure \(h_{d+1}\) with argument \(u_{d+1}\), pushing \((h_{d+1}, u_{d+1}, 0)\) onto the stack; or (3) perform an action \(a_t\), setting \(m_t\) to the state of the call-stack at that point. The counter \(\tau_d\) advances for each non-terminating PHP step. PHPs generalize nested options (Sutton et al., 1999) by allowing their operation to depend on \(u\) and \(\tau\).

We represent each PHP as a differentiable parametric model, outputting the log-probability of the PHP step \(\log \pi^h(\text{operation}|u, \tau, o)\). The log-probability of each time step \(\log p_{\theta}(m_t, a_t|m_{t-1}, o_t)\) breaks down into the sum...
of log-probabilities of the PHP steps (pops and pushes) that transition the call-stack from \( m_{t-1} \) to \( m_t \). For strongly supervised demonstrations \((\zeta, \xi)\), we can thus use supervised learning to maximize the log-likelihood

\[
\log p_{\theta}(\zeta, \xi) = \sum_{t=0}^{T-1} \log p_{\theta}(m_t, a_t|m_{t-1}, a_{t-1}) + \text{const.} \quad (1)
\]

In weak supervision, where \( \zeta \) is latent, we propose an amortized SVI method that replaces the log-likelihood \( \log p_{\theta}(\zeta|\xi) \) with its evidence lower bound (ELBO)

\[
\mathbb{E}_{\zeta|x_{=q_{\phi}}} \left[ \log \frac{p_{\theta}(\zeta, \xi)}{q_{\phi}(\zeta|\xi)} \right], \quad (2)
\]

where \( q_{\phi}(\zeta|\xi) \) is an inference network that approximates the computationally infeasible posterior \( p_{\theta}(\zeta|\xi) \) induced by the generator network, i.e. the actual PHPs.

We propose an architecture for the inference network \( q_{\phi} \) that extends SRNNs (Fraccaro et al., 2016) to support our hierarchical structure. We start by concatenating each observation–action pair \((a_t, a_t)\) in \( \xi \), and feeding this sequence into a bidirectional RNN. The output \( b_t \) of the RNN at every time step is a posterior context — a sequence in which each element is a function of the entire trace \( \xi \), allowing the decomposition \( q_{\phi}(\zeta|\xi) = \prod_{t=0}^{T-1} q_{\phi}(m_t|m_{t-1}, b_t) \).

Using the posterior context, we proceed to define \( q_{\phi}(m_t|m_{t-1}, b_t) \) similarly to \( p_{\theta}(m_t, a_t|m_{t-1}, a_{t-1}) \), as a product of PHP steps, except that posterior PHPs \( \pi_{\phi}(\text{operation}|u, \tau, b_t) \) are used instead of the usual PHPs \( \pi_{\theta}(\text{operation}|u, \tau, a_t) \). Since each transition is conditioned on the true action taken in that time step, posterior PHPs have structural constraints on their allowed outputs, enforced via masking the output logits (before log_softmax normalization), namely: only ancestors of the true action in the call-graph can be called; the root PHP cannot terminate before the final time step; and all PHPs must terminate at the final time step.

SVI estimates the ELBO (2) by sampling \( \zeta \) from the inference network \( q_{\phi} \) and computing the log probability ratio of \( p_{\theta} \) to \( q_{\phi} \). We minimize this loss by stochastic gradient descent on \( \theta \) and \( \phi \). To allow both sampling and gradients of \( q_{\phi} \), we use the relaxed one-hot categorical distribution, i.e. apply softmax to the logits after adding independent Gumbel variables (Jang et al., 2016; Maddison et al., 2016). Sampling is facilitated by the straight-through approximation, i.e. using argmax for samples and softmax for log-probabilities and gradient backpropagation. Note that this prevents gradient backpropagation through PHP steps.

**Experiment 1: MNIST Elevator.** We evaluate our method on a new benchmark called **MNIST Elevator**, designed to test the ability of PHPs to learn to generate arguments for sub-procedures. The root procedure, **elevator**, receives as argument the target floor as an MNIST digit (Le-Cun, 1998). It should then pass the one-hot decoded digit to a navigate procedure, which decides based on the current floor, observed as another MNIST digit, whether to go up or down. The main challenge in this domain is to train convolutional neural networks — namely, the navigate procedure in strong supervision and both procedures in weak supervision — to classify MNIST digits from only the order relation between two digits.

We found that pre-training with only strong supervision is needed to prevent the weak supervision from obscuring the strong supervision signal. We therefore triggered a late onset of the weak supervision signal, adding it to the mixture after 50000 trajectory samples (Figure 1). This immediately showed a significant improvement in the error rate, i.e. the fraction of imperfectly reproduced test traces, matching that of a larger strongly supervised dataset.

**Experiment 2: Karel.** Karel is an educational programming language (Pattis, 1981; Devlin et al., 2017; Bunel et al., 2018), generating sequences of actions for a robot in a grid world. Each cell in the grid can contain either a wall, or between 0 and 10 markers. The robot can move (forward), turnLeft, turnRight, pickMarker, or putMarker. The observations consist of leftIsClear, rightIsClear, frontIsClear, and markersPresent.

Each Karel program has its own hierarchical structure. We provide strong supervision by automatically parsing a given Karel program to create one PHP for the top-level function, and one for each control-flow construct in the program.

Table 1 summarizes our results on three textbook Karel programs. Overall, we see a benefit from training on more weakly supervised traces. Notably, for programs A and B, using 8 strongly supervised demonstrations out of 64 total demonstrations achieved similar results to having all 64 demonstrations strongly supervised, showing effective learning from weakly supervised demonstrations.
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